Nucleon-nucleon cross sections in neutron-rich matter

F. Sammarruca and P. Krastev
Physics Department, University of Idaho, Moscow, ID 83844, U.S.A

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We calculate nucleon-nucleon cross sections in the nuclear medium with unequal densities of protons and neutrons. We use the Dirac-Brueckner-Hartree-Fock approach together with realistic nucleon-nucleon potentials. We examine the effect of neutron/proton asymmetry and find that, although generally mild, it can be significant in specific regions of the phase space under consideration.

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I. INTRODUCTION

A topic of contemporary interest in nuclear physics is the investigation of the effective nucleon-nucleon (NN) interaction in a dense hadronic environment. Such environment can be produced in the laboratory via energetic heavy ion (HI) collisions or can be found in astrophysical systems, particularly the interior of neutron stars. In all cases predictions rely heavily on the nuclear equation of state (EOS), which is one of the main ingredients for transport simulations of HI collisions as well as the calculation of neutron star properties.

Transport equations, such as the Boltzmann-Uehling-Uhlenbeck (BUU) equation, describe the evolution of a non-equilibrium gas of strongly interacting hadrons. In BUU-type models, particles drift in the presence of the mean field while undergoing two-body collisions, which require the knowledge of in-medium two-body cross sections. In a microscopic approach, both the mean field and the binary collisions are calculated self-consistently starting from the bare two-nucleon force.

We present microscopic predictions of NN cross sections in symmetric and asymmetric nuclear matter. In asymmetric matter, the cross section becomes isospin dependent beyond the usual and well-known differences between the np and the pp/nn cases. It depends upon the total density and the degree of proton/neutron asymmetry, which of course also implies that the pp and the nn cases will in general be different from each other. In this paper, we will only be concerned with the the strong interaction contribution to the cross section (Coulomb contributions to the pp cross section are not considered).

Asymmetry considerations are of particular interest at this time. The planned Rare Isotope Accelerator will offer the opportunity to study collisions of neutron-rich nuclei which are capable of producing extended regions of space/time where both the total nucleon density and the neutron/proton asymmetry are large. Isospin-dependent BUU transport models [1] include isospin-sensitive collision dynamics through the elementary pp, nn, and np cross sections and the mean field (which is now different for protons and neutrons). The latter is a crucial isospin-dependent mechanism and is the focal point of an earlier paper [2].

In a simpler approach, the assumption is made that the transition matrix in the medium is approximately the same as the one in vacuum and that medium effects on the cross

section come in only through the use of nucleon effective masses in the phase space factors [3,4]. Concerning microscopic calculations, earlier predictions can be found, for instance, in Refs. [5,6], but asymmetry considerations are not included in those predictions. On the other hand, it is important to investigate to which extent the in-medium cross sections are sensitive to changes in proton/neutron ratio, the main purpose of this paper. In-medium cross sections are necessary to study the mean free path of nucleons in nuclear matter and thus nuclear transparency. The latter is obviously related to the total reaction cross section of a nucleus, which, in turn, can be used to extract nuclear r.m.s. radii within Glauber-type models [7]. Therefore, accurate in-medium *isospin-dependent* NN cross sections can ultimately be very valuable to obtain information about the size of exotic, neutron-rich nuclei.

In the next Section, after providing some details on the calculation, we present and discuss our results. Our conclusions are summarized in Section III.

II. IN-MEDIUM NN CROSS SECTIONS

A. General aspects

Details of our application of the Dirac-Brueckner-Hartree-Fock framework to asymmetric matter can be found in Ref. [8]. We choose the Bonn-B potential [9] as our model for the free-space two-nucleon (2N) force.

The nuclear matter calculation of Ref. [8] provides, together with the EOS, the single-proton/neutron potentials as well as their parametrization in terms of effective masses [2]. Those effective masses, together with the appropriate Pauli operator (depending on the type of nucleons involved), are then used in a separate calculation of the in-medium reaction matrix (or G-matrix) under the desired kinematical conditions.

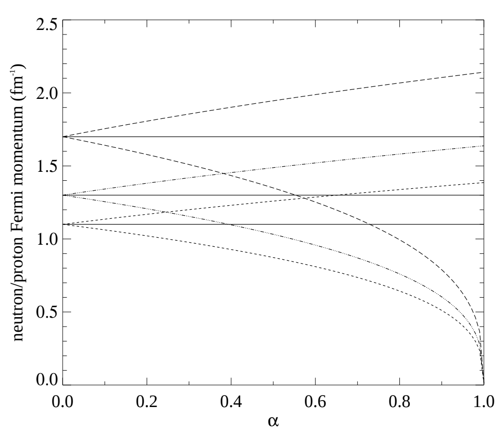

FIG. 1. Increasing(decreasing) of the neutron(proton) Fermi momentum as a function of the asymmetry parameter. The average Fermi momenta corresponding to the three groups of curves are 1.1, 1.3, and 1.7 fm^{-1}, respectively.

Our calculation is controlled by the total density, ρ, and the degree of asymmetry, $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, with ρ_n and ρ_p the neutron and proton densities. For the case of identical nucleons, the G-matrix is calculated using the appropriate effective mass, m_i, and the appropriate Pauli operator, Q_{ii}, depending on k_F^i, where $i = p$ or n. For non-identical nucleons, we use the "asymmetric" Pauli operator, Q_{ij}, depending on both k_F^n and k_F^p [8]. In Fig. 1, we show the variations of k_F^n and k_F^p with increasing neutron fraction at three fixed densities, according to the relations

$$k_F^n = k_F(1+\alpha)^{1/3} \tag{1}$$

$$k_F^p = k_F(1-\alpha)^{1/3}. \tag{2}$$

This may facilitate the interpretation of results later on.

In the usual free-space scattering scenario, the cross section is typically represented as a function of the incident laboratory energy, which is uniquely related to the nucleon momentum in the two-body c.m. frame, q_0 (also equal to the relative momentum of the two nucleons), through the well-known formula $T_{lab} = 2q_0^2/m$. In nuclear matter, though, the Pauli operator depends also on the total momentum of the two nucleons in the nuclear matter rest frame. This could be defined in some average (density-dependent) manner, or kept as an extra degree of freedom on which the cross section will depend. We will take the latter approach and examine this aspect in the next subsections.

Another issue to consider carefully when approaching the concept of in-medium cross sections is the non-unitary nature of the interaction. Due to the presence of Pauli blocking, the in-medium scattering matrix does not obey the free-space unitarity relations through which phase parameters are usually defined and from which it is customary to determine the NN scattering observables. As done in Ref. [6] but unlike Ref. [5], we calculate the cross section directly from the scattering matrix elements thus avoiding the assumption of in-vacuum unitarity and its consequences (such as the optical theorem). That is, we integrate the differential cross section

$$\sigma(q_0, \rho) = \int \frac{d\sigma}{d\Omega}(q_0, \theta, \rho) d\Omega, \tag{3}$$

where $\frac{d\sigma}{d\Omega}$ contains the usual sum of amplitudes squared and phase space factors. An alternative way would be to include Pauli blocking in the definition of phase parameters [10].

B. Results for pp and nn cross sections

As a baseline, we start by showing the pp cross section as a function of the momentum q_0 and at different densities of symmetric matter, see Fig. 2. Until otherwise noted, we use in-vacuum kinematics to define the total 2N momentum in the nuclear matter rest frame (that is, the target nucleon is, on the average, at rest). The range of momentum in Fig. 2 corresponds to values of the in-vacuum laboratory energy between approximately 20 and 260 MeV.

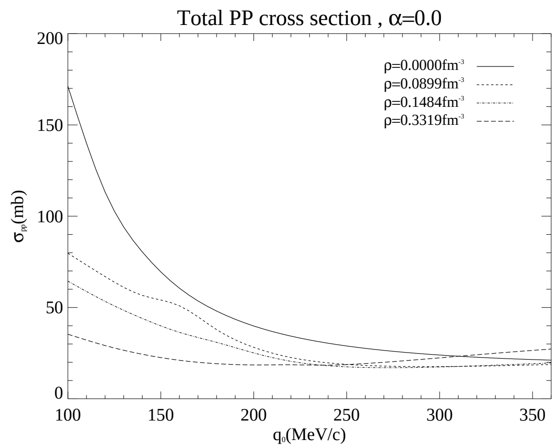

FIG. 2. Total pp cross section in symmetric matter as a function of the momentum in the 2N c.m. frame at the densities indicated in the figure.

As compared to the predictions of Ref. [5], where the use of the K-matrix approach imposes unitarity of the in-medium scattering matrix, the suppression of the cross section with increasing density is less pronounced. This was already pointed out in Ref. [6] and is most likely due to the different handling of the unitarity issue as noted above. Overall, we find reasonable qualitative agreement with previous predictions. We also notice that the high-density cross section tends to rise again at large momenta, a behavior already observed in Refs. [5,6] particularly for the pp channel.

A simpler approach to calculating in-medium cross sections consists of scaling the free-space value according to the relation [4]

$$\sigma(q_0, \rho) = (m^*(\rho)/m)^2 \sigma_{free}(q_0) \tag{4}$$

where m^* is the effective mass. In Fig. 3 we compare this approximation with the predictions from our full calculation. We see that at low momentum, and particularly at the lower densities, the agreement is quite good, but it strongly deteriorates at higher momenta and densities. Medium effects on the interaction are clearly important, particularly the interplay between the (energy-dependent) Pauli blocking mechanism and the dressing of the quasiparticle due to the surrounding medium.

We next examine the role of isospin asymmetry, the focal point of this paper. In Fig. 4, the ratio of the in-medium cross section to the free-space one is shown for both pp and nn scatterings as a function of the asymmetry. The total density is fixed to its value near saturation and the momentum q_0 is chosen to be equal to the corresponding Fermi momentum (k_F=1.3 fm^{-1}). The dependence on α is generally weak, which may be attributed to the "competing" roles of the effective mass and Pauli blocking. For the pp case, for instance, the smaller effective mass [2] and the lower Fermi momentum (see Fig. 1) would tend to decrease and increase the cross section, respectively. The opposite happens in the nn case. For the momentum considered in Fig. 4, the role of the effective mass appears to be dominant, lowering the pp cross section and raising the nn one. This trend of the pp/nn cross sections versus α and relative to each other is in qualitative agreement with predictions based

on scaling the cross section as in Eq.(4), with effective masses obtained from the modified Gogny interation recently used in isospin-dependent BUU calculations [11].

Figure 5 displays the cross section ratios for fixed values of the asymmetry and density but changing relative momentum. There, we see that the ratio approachs one in the high-momentum limit.

In Fig. 6, the cross section ratios are shown as functions of the (average) Fermi momentum but fixed asymmetry and q_0. The latter is chosen as in Fig. 4. The cross sections are seen to go down rather quickly with increasing density but then rise again at the higher densities. As we mentioned earlier, this was already observed, particularly in the *pp* channel, in previous microscopic calculations of cross sections in symmetric matter [5,6]. In this region of the phase space, disagreement with predictions based on Eq. (4) [11] reflects the discrepancy already noted in the high-density part of the lower panel in Fig. 3.

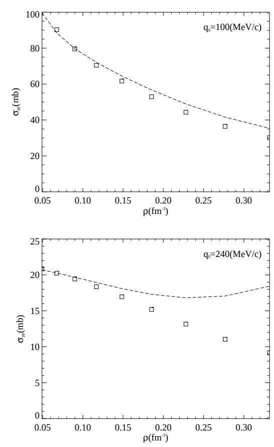

FIG. 3. Our calculated total *pp* cross section in symmetric matter compared with the one obtained from Eq. (4) (squares).

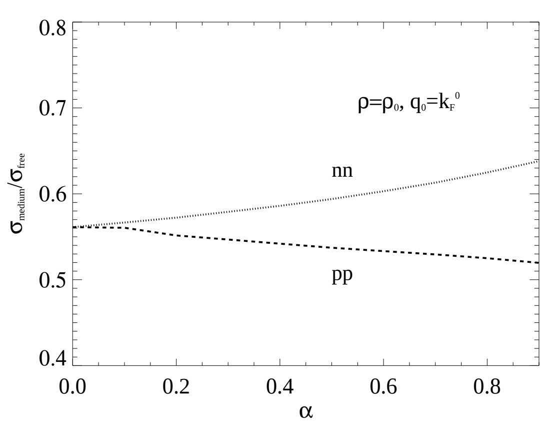

FIG. 4. Ratio of the total *pp* and *nn* cross sections to their free-space values as a function of the asymmetry near saturation density and fixed 2N relative momentum.

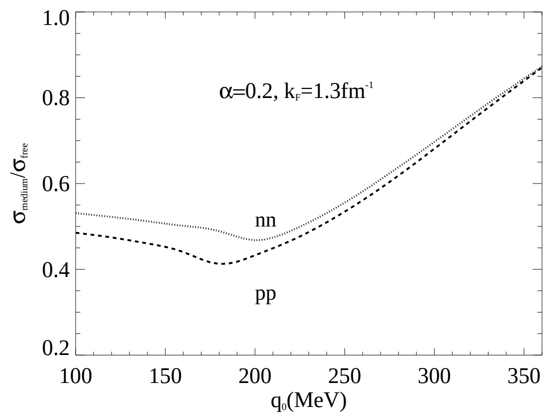

FIG. 5. Ratio of the total *pp* and *nn* cross sections to their free-space values versus the momentum q_0 at fixed asymmetry and total density.

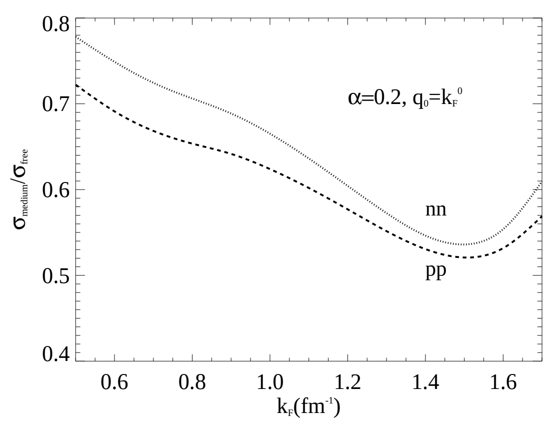

FIG. 6. Ratio of the total *pp* and *nn* cross sections to their free-space values versus the average Fermi momentum for fixed asymmetry and relative momentum.

Following up on the comments we made at the end of Subsection IIA, in the next figures we examine the impact of treating the 2N total momentum with respect to nuclear matter, P, as an independent variable (rather than linked to q_0 through the simplifying assumption of in-vacuum kinematics).

In Fig. 7, we show *pp* and *nn* cross section ratios under the same conditions of density and two-nucleon relative momentum as in Fig. 4 but changing value of the total 2N momentum, P. The general tendency seems rather independent of P, although some minor differences can be seen with Fig. 4 as well as among the three situations displayed in Fig. 7. In particular, we notice that for the lower values of P the *pp* cross section shows a lesser tendency to decrease with increasing α, and perhaps even a slight tendency to rise again. This may be understood in terms of Pauli blocking, which becomes more important at lower values of P and whose effect is opposite the one coming from the decreasing proton mass.

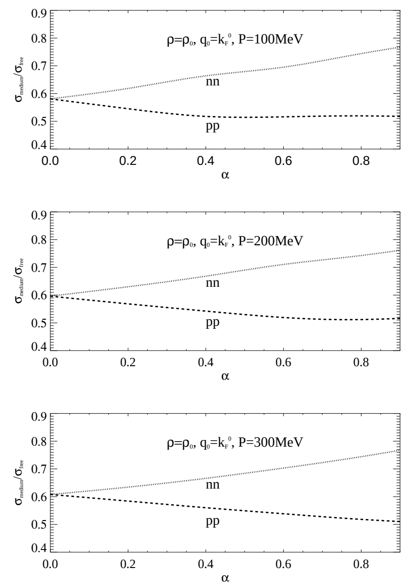

FIG. 7. Ratio of the total *pp* and *nn* cross sections to their free-space values as a function of the asymmetry at saturation density and fixed relative momentum. The value of the total momentum in the nuclear matter rest frame is indicated in each frame.

Next, to broaden our previous analysis, we show in Fig. 8 and Fig. 9 situations analogous to those presented in Fig. 4 and Fig. 7, respectively, but at a lower density. The chosen value of the average Fermi momentum, which is also taken as the value of q_0, corresponds to approximately one-half of saturation density. Sensitivity to the asymmetry appears stronger at lower density, as seen from the more pronounced differences between the *pp* and *nn* curves in both Fig. 8 and Fig. 9.

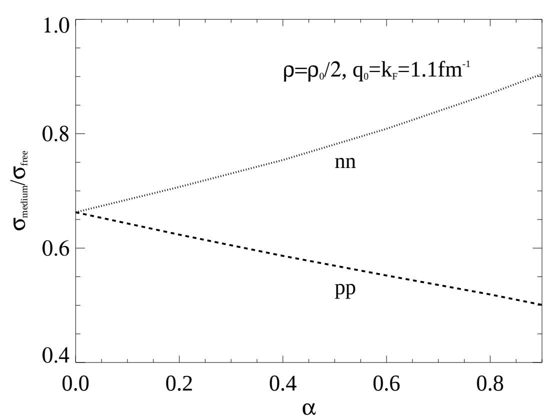

FIG. 8. Ratio of the total *pp* and *nn* cross sections to their free-space values as a function of the asymmetry at approximately one half of saturation density and fixed relative momentum.

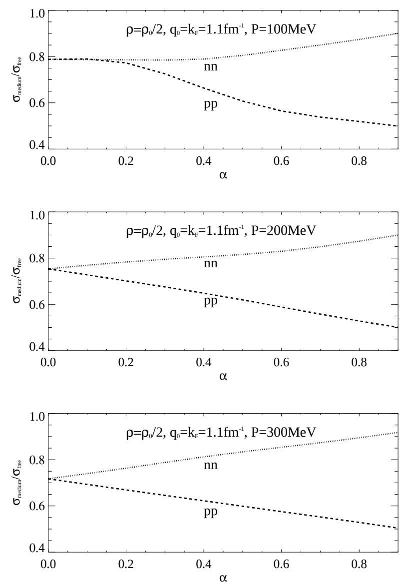

FIG. 9. Ratio of the total *pp* and *nn* cross sections to their free-space values as a function of the asymmetry at approximately one-half of saturation density and fixed relative momentum. The value of the total momentum in the nuclear matter rest frame is indicated in each frame.

C. Results for *np* cross sections

Similarly to what was done in Fig. 2 for the *pp* case, we begin with showing in Fig. 10 the basic density/momentum dependence of the *np* cross section in symmetric matter. (In-vacuum kinematics is used at this point.) We notice here that the cross section at the density equivalent to k_F=1.1 fm^{-1} shows some local enhancement which is considerably more pronounced than what can be seen in the *pp* case, see Fig. 2. This comparison suggests that enhanced attraction is generated at low density in the T=0 partial waves, most likely those with the quantum numbers of deuteron.

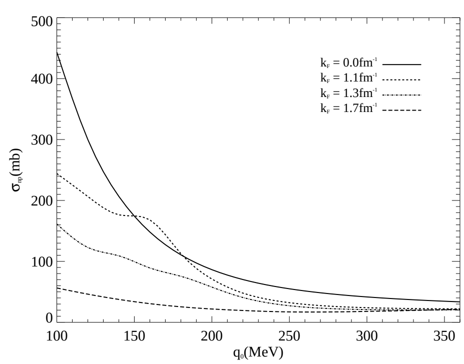

FIG. 10. Total *np* cross section in symmetric matter as a function of the momentum in the 2N c.m. frame at the densities indicated in the figure.

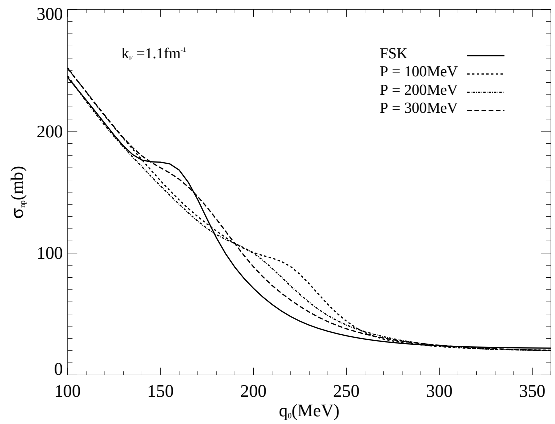

FIG. 11. Total np cross section in symmetric matter at approximately one-half of saturation density under different kinematical conditions. See text for details.

In Fig. 11, we explore more closely the dependence of these "structures" on the kinematics. The label "FSK" stands for free-space kinematics (as was used in Fig. 10), whereas the other three curves refer to different choices of the total momentum P as indicated in the figure. Clearly, the location of these enhancements is determined by the kinematics, moving to lower values of q_0 for higher values of P. Although noticible, these gentle fluctuations do not resemble at all the extremely large resonance-like peaks reported in some earlier works [10] and interpreted as precursor of a superfluid phase transition in nuclear matter at particular densities/momenta. Although crucial for generating those resonances in Ref. [10] was the inclusion of hole-hole scattering, other authors have reported large bound-state signatures even without hole-hole contributions [12]. Our predictions do not confirm those findings. Very mild fluctuations are predicted in Ref. [6], whereas no enhancements at all appear in the calculations of Ref. [5].

We will focus next on the asymmetry dependence of the np cross section. Having observed in the previous subsection that asymmetry effects on protons and neutrons tend to move the cross section in opposite directions, we may reasonably expect very mild dependence on α. This is indeed confirmed in Fig. 12 (where in-vacuum kinematics is used), and Fig. 13 (where, as before, the sensitivity to the choice of P is explored).

For in-medium np scattering, the nature of the cross section is the result of a subtle combination of multiple effects. These come from both the neutron and proton effective masses and the role of both the neutron and proton Fermi momenta in the expression of the (asymmetric) Pauli operator [8]. In the end, the behavior versus α is nearly flat, although a very slight tendency to rise can be identified, with the exception of the lower-P curve in Fig. 13.

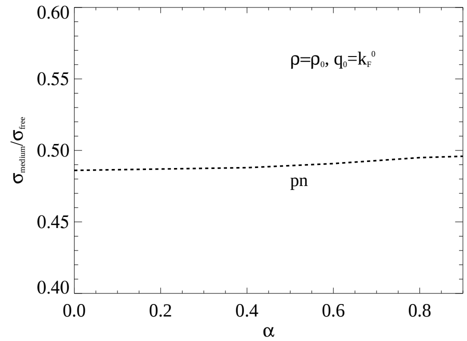

FIG. 12. Ratio of the total *np* cross sections to its free-space value as a function of the asymmetry at approximately saturation density and fixed relative momentum.

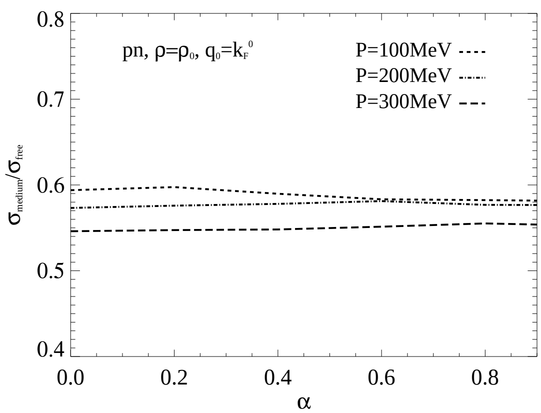

FIG. 13. Ratio of the total *np* cross section to its free-space value as a function of the asymmetry at saturation density and fixed relative momentum. The value of the total momentum in the nuclear matter rest frame is indicated in each frame.

III. CONCLUSIONS

We have presented microscopic calculations of cross sections for scattering of nucleons in neutron-rich matter. The sensitivity to the asymmetry in neutron/proton ratio comes through the combined effect of Pauli blocking and changing effective masses. The lowering(rising) of the proton(neutron) Fermi momentum and the reduced(increased) proton(neutron) effective mass tend to move the cross section in opposite directions. In the phase space regions we have examined here, the general tendency is for the *pp* cross section to go down and the *nn* one to rise with increasing asymmetry. This indicates that the role of the effective mass is dominant over Pauli blocking.

When both kinds of nucleons are involved, though, dispersive and Pauli blocking effects on each type of nucleon result in large cancelations. The np cross section is essentially α-independent, although minor variations can be seen depending on the kinematics.

On the other hand, the basic density/momentum dependence of the np cross section in symmetric matter appears rather interesting. Different predictions disagree concerning the presence of large bound-state effects which might be associated with the onset of superfluidity.

In summary, sensitivity to the asymmetry can be non-negligible for scattering of identical nucleons. The degree of sensitivity depends on the region of the energy-density-asymmetry phase space under consideration. We conclude that the proton mean free path could be affected in a significant way by in-medium scattering, mostly through σ_{pp}.

To test these findings, it is very useful to identify HI collision observables specifically sensitive to the two-body cross sections. Possible probes of the isospin-dependence of the in-medium NN cross sections are presently being investigated [13]. Whereas traditional means such as measurements of the stopping power are found to be ambiguous with respect to determining isospin dependence, potential candidates include isospin tracers like the neutron/proton free-nucleon ratio in reactions induced by radioactive beams in inverse kinematics [13].

ACKNOWLEDGMENTS

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[1] See, for instance, B.A. Li, Phys. Rev. Lett. **85**, 4221 (2000), and references therein.

[2] F. Sammarruca, W. Barredo, and P. Krastev, Phys. Rev. C **71**, 064306 (2005).

[3] V.R. Pandharipande and S.C. Pieper, Phys. Rev. C **45**, 791 (1992).

[4] D. Persram and C. Gale, Phys. Rev. C **65**, 064611 (2002).

[5] G.Q. Li and R. Machleidt, Phys. Rev. C **48**, 1702 (1993); **49**, 566 (1994).

[6] C. Fuchs, A. Faessler, and M. El-Shabshiry, Phys. Rev. C **64**, 024003 (2001).

[7] See, for instance, R. Crespo and R.C. Johnson, Phys. Rev. C **60**, 034007, (1999), and references therein.

[8] D. Alonso and F. Sammarruca, Phys. Rev. C **67**, 054301 (2003).

[9] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[10] T. Alm, G. Ropke, and M. Schmidt, Phys. Rev. C **50**, 31 (1994).

[11] Bao-An Li, private communications.

[12] A. Bohnet, N. Ohtsuka, J. Aichelin, R. Linden, and A. Faessler, Nucl. Phys. **A494**, 349 (1989).

[13] B.-A. Li, P. Danielewicz, and W. Lynch, nucl-th/0503038.